Mathematics
Advanced
Paper 1: Pure Mathematics 1

Paper 1 Pure Mathematics 1	
You must have: Mathematical Formulae and Statistical Tables, calculator	
Time allowed	2 hours

Write all of you answers on lined A4 paper.

Make sure you write your name and your teacher's name at the top of every page.

Total marks	/100

1 Given that $(x + 2)$ and $(x - 1)$ are factors of $px^3 + 5x^2 + qx - 6 = 0$,

a find the values of p and q.

(4)

b Hence sketch the graph of $y = px^3 + 5x^2 + qx - 6$ labelling all points of intersection with the coordinate axes.

(4)

(Total for Question 1 is 8 marks)

2 A function is defined by,

$$\mathrm{f}(\theta) = 2 + 3\sec(\theta - 30°) \text{ for all values of } \theta$$

a State the range of values of the constant k for which $\mathrm{f}(\theta) = k$ has no solutions.

(2)

b Solve $\mathrm{f}(\theta) = 7$ in the interval $-180^\circ \leq \theta \leq 180^\circ$.

(4)

(Total for Question 2 is 6 marks)

3 The ninth term of an arithmetic series is 36.

The thirteenth term is 16.

The sum of the first n terms is 400.

a Show that $5n^2 - 157n + 800 = 0$

(5)

b Hence find the value of n.

(2)

(Total for Question 3 is 7 marks)

4 **a** Find $\int_{\frac{\pi}{6k}}^{\frac{\pi}{3k}} (2\pi \sec^2 kx + 2)\, \mathrm{d}x$, where k is constant, giving your answer in terms of k.

(4)

b Given $\int_{\frac{\pi}{6k}}^{\frac{\pi}{3k}} (2\pi \sec^2 kx + 2)\, \mathrm{d}x = \pi(8\sqrt{3} + 2)$, find the exact value of k.

(1)

(Total for Question 4 is 5 marks)

5 Environmentalists are modelling the number of people, N, in thousands, affected by a nuclear meltdown t hours after the meltdown occurs.

The line l shown in Figure 1 illustrates the linear relationship between t and $\log_{10} N$ for the first 20 hours.

The line l meets the vertical axis at (0, 2.1), as shown.

The gradient of l is0.1.

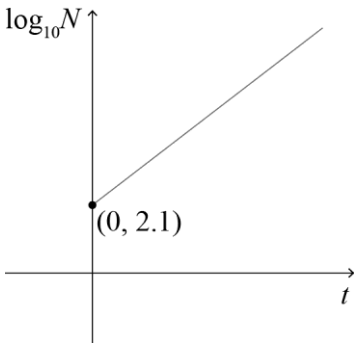

Figure 1

a Write down an equation for l.

(2)

The environmentalists wish to write the relationship between N and t in the form $N = ab^t$.

b Find the value of a and the value of b correct to 3 significant figures.

(4)

c With reference to the model, interpret the value of a and the value of b.

(2)

d Find the population affected, to the nearest 1000, by the model when $t = 15$.

(1)

e Find the number of hours it takes for the population afftected to reach 8 000 000.

(2)

f State one reason why this may not be a realistic model.

(1)

(Total for Question 5 is 12 marks)

6 Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = a - 2|x - 6|, \ x \in \mathbb{R}$$

The graph intercepts the y-axis at $(0, -8)$.

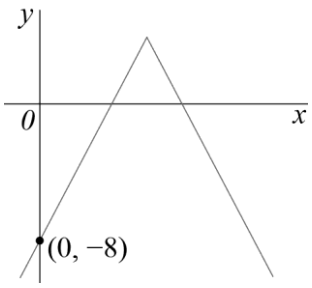

Figure 2

a Find the value of a.

(1)

b Solve $f(x) \geq -\frac{1}{2}x + 6$

(4)

(Total for Question 6 is 5 marks)

7 **a** Prove that,

$$sec^2\theta + cosec^2\theta \equiv 4cosec^2 2\theta, \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

b Explain why the equation $\sec^2\theta + \operatorname{cosec}^2\theta = 1$ does not have any real solutions.

(2)

(Total for Question 7 is 6 marks)

8 Figure 3 shows the curve with equation,

$$y = sin^2x\ cos^3x, 0 \leq x \leq \frac{\pi}{2}$$

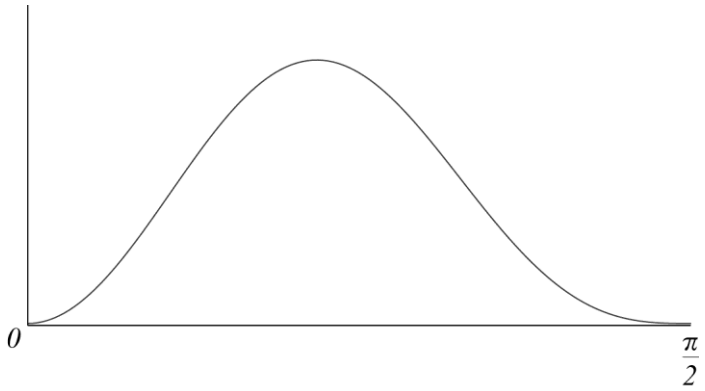

Figure 3

a Giving your answers to 4 significant figures, complete the table with the value of *y* corresponding to $x = \frac{2\pi}{5}$.

(1)

x	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$	$\frac{3\pi}{10}$	$\frac{2\pi}{5}$	$\frac{\pi}{2}$
y	0	0.0821	0.1829	0.1329		0

b Given that $I = \int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx$,

i use the trapezium rule with five strips to find an approximate value for *I*, *g*iving your answer to 4 significant figures.

(3)

ii Explain how the trapezium rule could be used to obtain a more accurate estimate of the integral.

(1)

c By using an appropriate substitution, or otherwise, find the exact value of *I*.

(6)

(Total for Question 8 is 11 marks)

9 A function is defined by,

$$f(x) = 2(x-4)^2 - 6 - e^{1-0.5x}, x \geq 0$$

a Show f (x) = 0 has a root α in the interval [5.7, 5.8].

(2)

b A student takes 5.7 as a first approximation to α.

Given f'(5.7) = 6.8786 to 5 significant figures, apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 decimal places.

(2)

c Sketch a graph to show that there are exactly two roots of f (x) = 0.

(2)

(Total for Question 9 is 6 marks)

10 A student states, 'if $\frac{a}{b}$ is an irrational number, then at least one of a and b is an irrational number'.

a Use proof by contradiction to prove that the student is correct.

(4)

He also states, 'if $\frac{a}{b}$ is a rational number, then both a and b are rational numbers.'

b Show that this statement is not true.

(1)

(Total for Question 10 is 5 marks)

11 Figure 4 shows rectangle $ACDE$ with $\angle DAB = \theta,\ \angle DBC = 2\theta$ and $AB = BD = 1$.

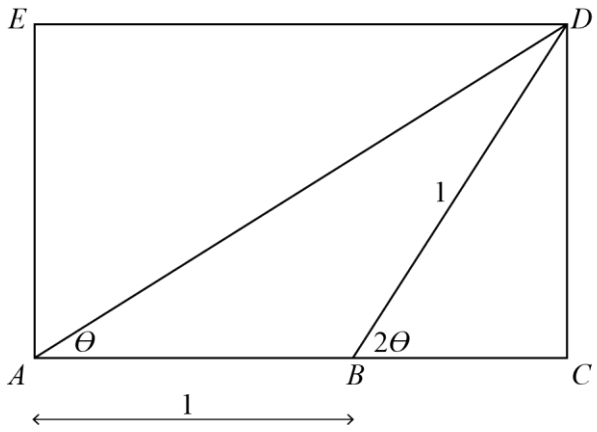

Figure 4

a Show that $AD = \sqrt{2\cos 2\theta + 2}$.

(3)

b Hence prove that $\cos 2\theta = 2\cos^2 \theta - 1$.

(3)

(Total for Question 11 is 6 marks)

12 The curve C has parametric equations,

$$x = 2\sin t,\ y = \frac{1}{2}cosec\ t, 0 \leq t \leq 2\pi$$

a Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{\pi}{4}$. The line l is normal to C at P.

b Show that the equation for l is $y = 2x - \frac{3\sqrt{2}}{2}$.

(5)

The line l intersects the curve C again at Q.

c Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

(Total for Question 12 is 13 marks)

13 Figure 5 shows a cyclindrical-shaped barrel of diameter 40 cm. It holds rain water.

Rain water flows out of the tap near the bottom of the barrel at a rate proportional to the square root of the volume.

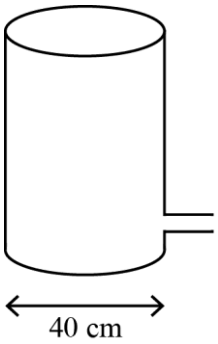

Figure 5

a Show that, at t seconds after the tap is opened, $\frac{\mathrm{d}h}{\mathrm{d}t} = -k\sqrt{h}$ for some constant k.

(4)

b Show that the general solution to this differential equation is $h = (A - Bt)^2$.

(3)

Initially, the height of the water is 64 cm. 30 seconds later, the height of the water is 25 cm.

c Find the value of the constants A and B.

(2)

d Explain why it might not be appropriate to use the model to calculate the time when the height of the water is 1 cm.

(1)

(Total for Question 13 is 10 marks)

TOTAL FOR PAPER IS 100 MARKS