Mathematics

Advanced

Paper 2: Pure Mathematics 2

Paper 2 Pure Mathematics 2	
You must have: Mathematical Formulae and Statistical Tables, Calculator	
Time allowed	2 hours

Write all of you answers on lined A4 paper.

Make sure you write your name and your teacher's name at the top of every page.

Total marks	/100

1

$$f(x) = \frac{1}{\sqrt{1-4x}}, \quad |x| < k$$

where k is a rational number.

a Find the series expansion of f(x), in ascending powers of x, up to and including the x^3 term.

(3)

b State the value of k.

(1)

(Total for Question 1 is 4 marks)

2 The line l_1 passes through the points $P(5, 8)$ and $Q(15, 16)$.

The line l_2 is perpendicular to l_1 and passes through the point P.

The line l_1 intersects the y-axis at A and the line l_2 intersects the y-axis at B.

Find the area of the triangle APB.

(8)

(Total for Question 2 is 8 marks)

3 A circle has equation $x^2 + 6x + y^2 - 4y - 32 = 0$.

a Show that the radius of the circle is of the form $a\sqrt{5}$, where a is a constant to be found.

(2)

b Find the coordinates of the centre of the circle.

(1)

c The points $A(-9, -1)$ and $B(3, 5)$ both lie on the circumference of the circle.

Show that AB is the diameter of the circle.

(2)

d The point D lies on the negative y-axis and the angle $ADB = 90°$.

Find the coordinates of D.

(3)

(Total for Question 3 is 8 marks)

4 A basketball player throws a basketball.

The height, h metres, of the basketball above the ground is modelled by the formula,

$$h = 1.9 + 2x - \frac{1}{3}x^2, x \geq 0$$

where x is the horizontal distance of the basketball from the player, measured in metres.

The basketball travels in a vertical plane until it either hits the hoop or the ground.

a With reference to the model, interpret the significance of the constant 1.9 in the formula.

(1)

b Write $1.9 + 2x - \frac{1}{3}x^2$ in the form $A - B(x - C)^2$, where A, B and C are constants to be found.

(3)

c State the greatest height of the basketball and the horizontal distance when it occurs.

(2)

(Total for Question 4 is 6 marks)

5 Find the exact solutions to the equation $2e^x + 15e^{-x} = 13$.

You must show clearly how you obtained your answer.

(4)

(Total for Question 5 is 4 marks)

6 The functions p and q are defined by,

$$\mathrm{p}(x) = |3 - 2x|$$
$$\mathrm{q}(x) = \frac{4x - 1}{5}$$

Solve $\mathrm{pq}(x) = \frac{3}{2}x$, giving your answers in exact form.

(5)

(Total for Question 6 is 5 marks)

7 The curve C has parametric equations,

$$x = \frac{4}{t} - 1,\ y = 2t + \frac{6}{t} - 8,\ t \neq 0$$

Show that the Cartesian equation of the curve can be written in the form,

$$y = \frac{(ax-1)(x-b)}{c(x+1)},\ x \neq -1$$

where a, b and c are constants to be found.

(5)

(Total for Question 7 is 5 marks)

8 Show that $\int_2^4 \frac{2x^2 + x - 9}{x^2 + x - 2}\,dx = 4 - \ln 6$

(7)

(Total for Question 8 is 7 marks)

9 The population of lions in a national park was 2400 at the beginning of 2008.

A model predicts that the lion population will decrease by 2% each year.

The model predicts that after n years, the population will be first below 1600.

a Show that $n > \dfrac{\log(\frac{2}{3})}{\log(\frac{49}{50})}$

(4)

A conservation agency wishes to raise £1000 per lion per year based on the number of lions predicted to be in the park at the start of each year.

b Calculate the total amount raised, to the nearest £1000, between the beginning of 2008 and the end of 2016.

(3)

c Explain why this model of fundraising might not be effective.

(1)

(Total for Question 9 is 8 marks)

10 Figure 1 shows a circle with centre O and radius r.

The chord AB divides the circle into a minor segment R_1 and a major segment R_2.

The chord AB subtends an angle Θ at O.

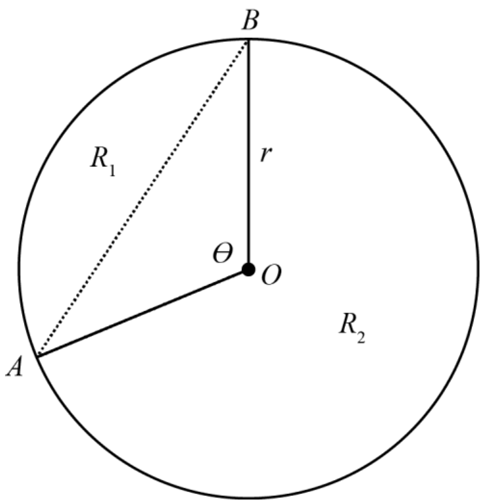

Figure 1 [not to scale]

a Show that the area of R_1 is $\frac{1}{2}r^2(\theta - \sin\theta)$.

(2)

b Given that the ratio of the areas of R_1 to R_2 is 1:9, show that $\sin\theta = \theta - \frac{1}{5}\pi$.

(3)

(Total for Question 10 is 5 marks)

11 Figure 2 shows curve C with equation $y = \mathrm{f}(x)$

where $\mathrm{f}(x) = \dfrac{\sin^2 x}{e^{\frac{1}{2}x}}$, $0 \leqslant x \leqslant 4$

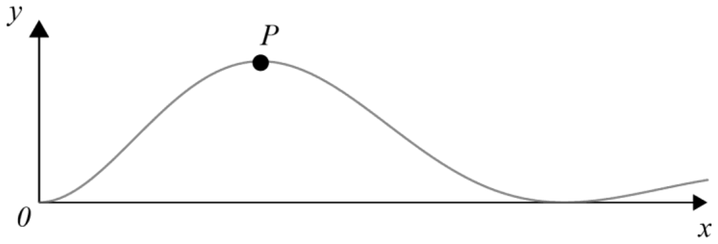

Figure 2

The curve has a local maximum turning point at P.

a Show that the x-coordinate of point P is a solution of the equation $\tan x = 4$.

(6)

b Using your answer to part **a**, find the x-coordinate of the maximum turning point on $y = 1 + 4\mathrm{f}(2x)$.

(2)

(Total for Question 11 is 8 marks)

12 The points A, B and C, with position vectors $(-5, 4, 7)$, $(-15, 8, 12)$ and $(-13, 12, 1)$ respectively, form a triangle ABC.

a Show that triangle ABC is isosceles.

(2)

b Calculate the size of angle ABC.

(2)

(Total for Question 12 is 4 marks)

13 a Show that, when θ is small, the equation $\dfrac{2\cos 4\theta + 4\tan 2\theta + 1}{1 + 2\sin 2\theta}$ can be written as $3 - 4\theta$.

(3)

b Hence write down the value of $\dfrac{2\cos 4\theta + 4\tan 2\theta + 1}{1 + 2\sin 2\theta}$ when θ is small.

(1)

(Total for Question 13 is 4 marks)

14 Prove by contradiction that $\sqrt{2}$ is an irrational number.

(5)

(Total for Question 14 is 5 marks)

15 The value of a car can be modelled by the function,

$$V = \frac{300}{18\sin\left(\frac{t}{16}\right) + 7\cos\left(\frac{t}{16}\right)}, 0 \leq t \leq 25$$

where,

V is the value of the car in thousands

t is the time in years since the car was purchased.

a Express $18\sin\theta + 7\cos\theta$ in the form $R(\sin\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the exact value of R and the value of α, in radians, to 2 decimal places.

(3)

b Find the time, in years, that it will take for the car to be worth half of its original purchase price.

(4)

c Find the time taken for the value of the car to reach its minimum value.

(2)

d Explain with a reason why this model is not valid for large values of t.

(1)

(Total for Question 15 is 10 marks)

16 The number of confirmed cases, D, per year, of a disease is decreasing. The rate of decrease is proportional to the number of confirmed cases per year.

a Given that initially the number of confirmed cases is D_0, show that $D = D_0e^{-kt}$

(5)

It is known that after 15 years there are only $\frac{1}{6}$ of the initial confirmed cases per year.

b Find the exact value of k.

(3)

c Will the disease ever be eliminated?

Explain your answer.

(1)

(Total for Question 16 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS