DHFS
Mathematics - Year 1
Paper 1: Pure and Statistics

Paper 1 Pure and Statistics	
You must have: mathematical formulae and statistical tables, calculator	
Time allowed	1 hour 45 minutes

Write all of you answers on lined A4 paper.

Make sure you write your name and your teacher's name at the top of every page.

Total marks	/86

SECTION A: Pure

Answer ALL questions

1 Find an equation of a line l which passes through $P(-2, 6)$ and $Q(4, -2)$.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

(Total for Question 1 is 3 marks)

2 Point P lies on the line with equation $2x - y - 5 = 0$.

Point P is a distance of $\sqrt{130}$ from the origin.

Show that there are two possible positions for point P and find the coordinates for each of these points.

Show each step of your working.

(5)

(Total for Question 2 is 5 marks)

3 The points $P(-5, -13)$ and $Q(7, 3)$ lie on a circle C with centre $(a, -8)$ and radius r.

Find the equation of the circle C.

(8)

(Total for Question 3 is 8 marks)

4 The equation $kx^2 - 3kx + 15 = 0$, where k is a constant, has two real roots.

Prove that $k < 0$ or $k > \frac{20}{3}$.

(3)

(Total for Question 4 is 3 marks)

5 Figure 1 shows a triangle, *ABC*.

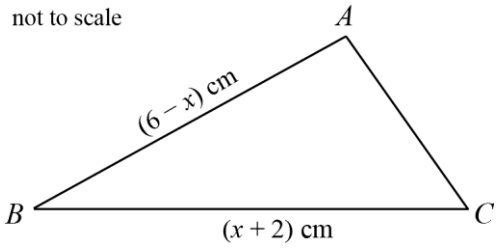

Figure 1

$\angle ABC = 30°$

$AB = (6 - x)$ cm

$BC = (x + 2)$ cm.

The area of the triangle is A cm^2.

a Show that $A = \frac{1}{4}\left(-x^2 + 4x + 12\right)$.

(3)

b Find the maximum value of A and the value of x at which it occurs.

(4)

(Total for Question 5 is 7 marks)

6 Solve for $-180 \leq x < 180$, $8\cos^2 x + 10\cos x = 13 - 5\sin^2 x$.

Give you're answers to one decimal place.

(5)

(Total for Question 6 is 5 marks)

7 Prove, from first principles, that the derivative of $4x^3$ is $12x^2$.

(4)

(Total for Question 7 is 4 marks)

8 $\overrightarrow{AB} = -3k\mathbf{i} + k\mathbf{j}$

The magnitude of $\overrightarrow{AB}$ is $5\sqrt{30}$

Find the possible values of k, leaving your answer in simplified surd form.

(3)

(Total for Question 8 is 3 marks)

9 Figure 2 shows a line with equation $x + y = 11$.

It intersects a curve with equation $y = -\frac{1}{2}x^2 + 4x + 3$ at the points P and Q.

The shaded region R_1 is a trapezium bounded by PQ, the x-axis and lines parallel to the y-axis through P and Q.

The shaded region R_2 is the finite region bounded by the line and the curve.

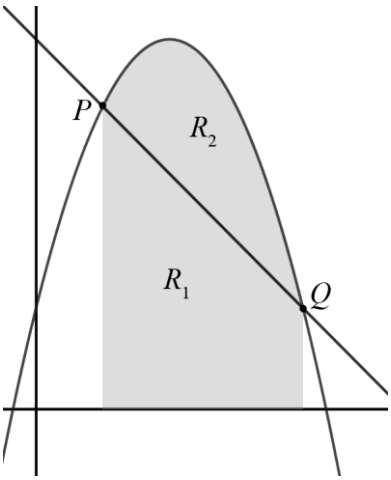

Figure 2

Show that the areas of the shaded regions R_1 and R_2 are in the ratio 2:1.

(8)

(Total for Question 9 is 8 marks)

10 Figure 3 shows the plan view of a garden where part of the garden has been enclosed with 250 m of fencing.

The shape of the enclosed part of the garden is a rectangular section joined to a semicircular section.

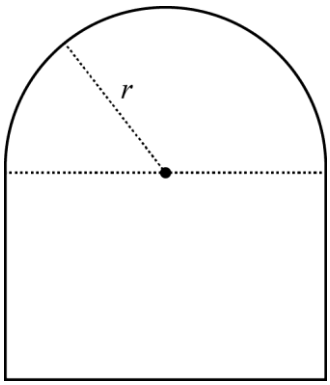

Figure 3

Given that the radius of the semicircular section is r metres, show that,

a the area, $A\,\text{m}^2$, of the enclosed part of the garden is given by $A = 250r - \left(\frac{4+\pi}{2}\right)r^2$

(5)

b the maximum value of the area of the enclosed part of the garden is $A = \frac{250^2}{2(4+\pi)}$

(5)

(Total for Question 10 is 10 marks)

TOTAL FOR SECTION A IS 56 MARKS

SECTION B: STATISTICS

Answer ALL questions

11 Briony investigated the variation in daily mean air temperature, x °C, for Beijing in May and June 2015.

She used the large data set to select a sample of size 15.

She selected the first value by generating a random number between 1 and 61 and then selected every fourth value after that.

a State the sampling technique that Briony used.

(1)

Briony summarised the data and found,

$$n = 15,\ \sum x = 339.6,\ \sum x^2 = 7994.8$$

b Calculate the standard deviation.

(2)

An outlier is defined as,

'a data value which is more than two standard deviations from the mean'.

c Show that the temperature recorded on 10 May, 9.7 °C, is an outlier.

(1)

d Clean the data and recalculate the mean and standard deviation for the new data set.

(3)

e From your knowledge of the large data set, explain why Briony's sampling process might not generate a sample of size 15.

(1)

(Total for Question 11 is 8 marks)

12 The Venn diagram, Figure 1, shows the probabilities for children at a nursery taking part in various activities.

P represents the event that a child takes part in finger painting.

Q represents the event that a child takes part in music.

R represents the event that a child takes part in water play.

x and *y* are probabilities.

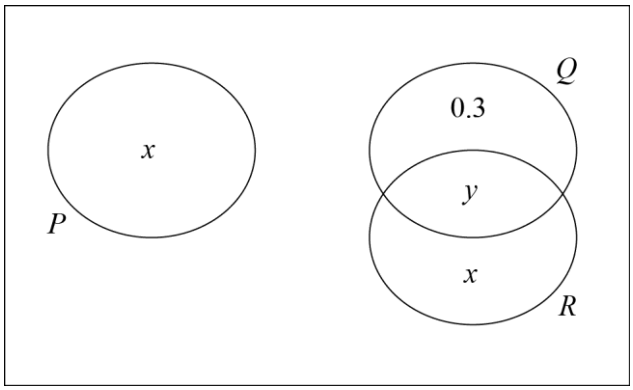

Figure 1

All the children take part in at least one activity.

The probability that they do not take part in music is 0.3

a Find the values of *x* and *y*.

(2)

b Find the probability that a randomly selected child takes part in music or painting.

(1)

c State, giving a reason, whether or not the events *Q* and *R* are statistically independent.

Show your working clearly.

(3)

(Total for Question 12 is 6 marks)

13 Historical data suggests that 20% of motorists regularly exceed the speed limit on a motorway.

A new law is introduced increasing the penalties for speeding and the police suspect that there has been a reduction in the number of motorists speeding.

A random sample of 30 motorists is taken.

a Write down the hypotheses that should be used to test the police's suspicion.

(1)

b Find the critical region for the test.

Use a significance level of 5%.

(2)

On a particular day, the police observe that three of the 30 motorists sampled exceeded the speed limit.

c Comment on the police's claim in the light of this observation.

(1)

It was later discovered that all the motorists in the sample were part of the same race club.

d Comment on the validity of the model used.

(1)

(Total for Question 13 is 5 marks)

14 A discrete random variable X is modelled using the probability function,

$$P(X = x) = kx^2, \ x = 1, 2, 3, 4$$

a Write down, in terms of k, $P(X = 2)$.

(1)

b Find the value of k.

(2)

Josh has a biased, four-sided dice that he claims can be modelled using the probability function given above.

He rolls the dice 300 times.

His results are shown in the table,

Score	1	2	3	4
Frequency	10	38	120	132

c Use the results in the table to find an estimate for the probability that Josh rolls a two.

(1)

d Comment on the suitability of the model.

(2)

(Total for Question 14 is 6 marks)

15 Ahmed is investigating the relationship between daily total rainfall (x mm) and daily total sunshine (y hours) in Leeming in July 2015.

a Describe the type of data represented by daily total rainfall.

(1)

Ahmed drew a scatter diagram for the data, as shown in Figure 2.

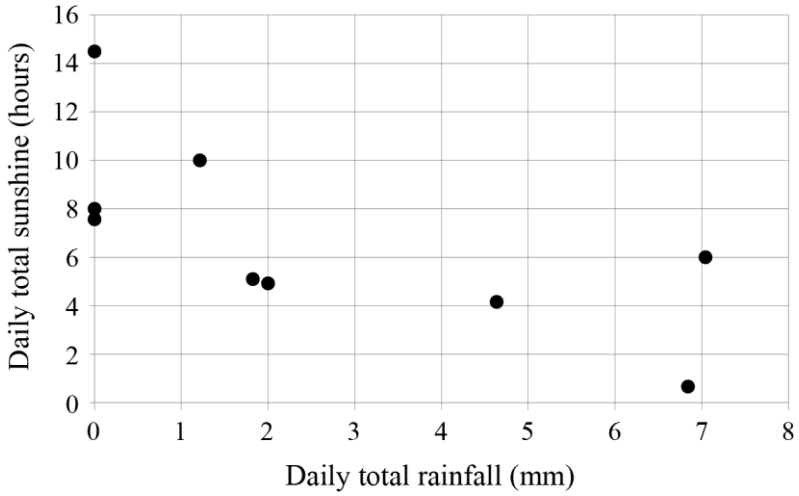

Figure 2

b Describe the type of correlation shown.

(1)

He calculates the equation of the regression line of y-on-x to be $y = 9.34 - 0.97x$.

c Give an interpretation of the value 9.34 in the equation of the regression line.

(1)

Ahmed had a tenth data value for total daily rainfall, but the total daily sunshine was missing.

d Evaluate the usefulness of outcome of Ahmed's model for the tenth day when the daily total rainfall was 10.1 mm.

(2)

(Total for Question 15 is 5 marks)

TOTAL FOR SECTION B IS 30 MARKS

TOTAL FOR PAPER IS 86 MARKS