DHFS
Mathematics - Year 1
Paper 2: Pure and Mechanics

Paper 2 Pure and Mechanics	
You must have: mathematical formulae and statistical tables, calculator	
Time allowed	1 hour 30 minutes

Write all of your answers on lined A4 paper.

Make sure you write your name and your teacher's name at the top of every page.

Total marks	/74

SECTION A: PURE

Answer ALL questions

1   A freelance journalist charges an initial fixed fee and then an extra fee per word.

   She charges £250 for a 300 word article and £410 for a 700 word article.

   a Write an equation linking words, w, and fee, F, in the form $F = aw + b$.

   (3)

   b Interpret the values of a and b.

   (2)

   She charges a company £650 to write another article.

   c Calculate the word length of this article.

   (2)

   (Total for Question 1 is 7 marks)

2 Given that $y = \dfrac{16}{81} x^4$, express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{3}{4}}$

(2)

b $\dfrac{2}{3} y^{-\frac{1}{2}}$

(2)

(Total for Question 2 is 4 marks)

3 A company expects to sell 20000 computers in the first year if the price of each computer is £650.

Let x represent the number of £'s by which the price has decreased.

a Write an expression for the price, p, of one computer, in the form $p = a + bx$.

(1)

The company expects to sell an additional 50 computers every time the price decreases by £1.

b Write an expression for the number of computers sold, C, in the form $C = d + ex$.

(1)

Revenue is defined by the formula,

revenue = (number of computers sold) × (cost of one computer)

c Write an equation for revenue, r, in the form $A - B(x - C)^2$, where A, B and C are constants to be found.

(4)

The company wishes to maximise the revenue.

d Using your answer to part **c**, or othwerwise, state the price the company should charge for each computer and the revenue they will attain.

(2)

(Total for Question 3 is 8 marks)

4 $f(x) = x^2 - 7x + 10$

$g(x) = 6 - 2x$

a Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same axes.

(4)

b Find the coordinates of any points of intersection.

(4)

c Write down the sets of values of x for which $g(x) > f(x)$.

(1)

(Total for Question 4 is 9 marks)

5 Prove that, for any positive numbers a and b, where $a \neq b$, $a^2 + b^2 > 2ab$.

(3)

(Total for Question 5 is 3 marks)

6   In the binomial expansion of $(1 + px)^8$, the coefficient of x^3 is 252 times the coefficient of x.

Find the value of the coefficient of x^2.

(5)

(Total for Question 6 is 5 marks)

7 The value, V in £'s, of a car t years after purchase can be modelled by the equation,

$$V = 28000e^{-0.19t} + 2000 \; for \; t \geq 0$$

a State the initial value of the car.

(1)

b Interpret the meaning of the 2000 in the model.

(1)

c Find $\dfrac{dV}{dt}$ and state how $\dfrac{dV}{dt}$ shows the value of the car decreases over time.

(2)

d Show that, when the value of the car is £18 000, $t = \dfrac{100}{19}\ln\left(\dfrac{7}{4}\right)$.

(4)

(Total for Question 7 is 8 marks)

TOTAL FOR SECTION A IS 44 MARKS

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

8 A remote-control car accelerates along a straight race track.

The velocity–time graph (Figure 3) shows the motion of the remote-control car over a period of 25 seconds. The points A and B on the graph have coordinates (5, 10) and (15, 10) respectively.
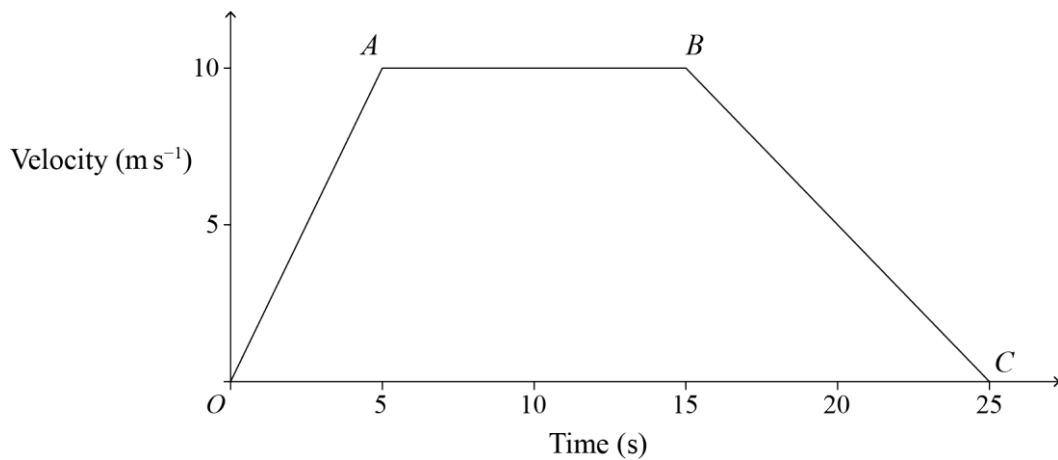

Figure 3

a Describe what is happening to the direction of travel of the remote-control car.

(1)

b Describe the motion of the car between,

i *O* and *A*

(1)

ii *A* and *B*.

(1)

c Calculate the total distance travelled by the car.

(2)

(Total for Question 8 is 5 marks)

9 A particle of mass 5 kg is at rest under the action of three forces,

$$F_1 = 8\mathbf{i} - 2\mathbf{j}, \ F_2 = a\mathbf{i} + b\mathbf{j} \text{ and } F_3 = a\mathbf{i} - \mathbf{j}$$

a Find the values of the constants a and b.

(3)

The direction of the force F_1 is reversed.

b Find the acceleration of the particle. Give your answer in vector form.

(3)

(Total for Question 9 is 6 marks)

10 A train engine of mass 12 tonnes is pulling a carriage of mass 6 tonnes along a horizontal track.

The engine produces a constant driving force of 10 kN.

The resistance to motion is 5000 N on the engine and 3000 N on the carriage.

The engine and the carriage are connected by a shunt.

a Show that the train is accelerating at $\frac{1}{9}$ m s^{-2}.

(3)

When the train is travelling at 30 m s^{-1}, the shunt fails.

b Work out how long it takes the carriage to come to a halt.

(3)

c State one modelling assumption you have made in answering part **b**.

(1)

(Total for Question 10 is 7 marks)

11 A ball B moves along a straight line.

B is initially at rest at the point O.

At time t s, the velocity of B is $t(1 - t^2)$ m s^{-1}.

Show that the maximum velocity of the ball is $\dfrac{2}{3\sqrt{3}}$ m s^{-1}.

(5)

(Total for Question 11 is 5 marks)

12 A ball is dropped from rest from a height of h metres onto a horizontal surface.

After striking the floor, it rebounds to half its original height.

Show that the time, t, taken from the instant the ball is dropped until the instant it strikes the floor for the second time is given by,

$$t = \left(2 + \sqrt{2}\right)\sqrt{\frac{h}{g}}$$

where g is the acceleration due to gravity.

(7)

(Total for Question 12 is 7 marks)

TOTAL FOR SECTION B IS 30 MARKS

TOTAL FOR PAPER IS 74 MARKS